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Comparison and implementation of the various numerical methods used for calculating transmission loss in silencer systems

S. Bilawchuk, K.R. Fyfe*

4-9 Mechanical Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2G8

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Abstract

Issues concerning the design and use of large-scale silencers are more prevalent today then ever before. With the increased use of large industrial machinery (such as gas turbines) and the increase in public awareness and concern for noise control, the desire to be able to properly design silencers for specific applications is increasing. Even today, most silencer design is performed by simply modifying existing designs without full confidence of the new performance characteristics. Due to the size and expense of these silencers, it would be beneficial to have means to predict the insertion loss (*IL*) or transmission loss (*TL*) characteristics at the design stage. To properly accomplish this, many factors such as geometry, absorptive material properties, flow effects, break out noise, and self-generated noise must be considered. The use of the finite element method (FEM) and the boundary element method (BEM) can aid in the prediction and design. This paper examines three of the different methods used in calculation of *TL* values; namely the "traditional" laboratory method, the 4-pole transfer matrix method and the 3-point method. A comparison of these methods based on such criteria as accuracy, computation time, and ease of use was conducted. In addition, the idiosyncrasies and problems encountered during implementation are presented. The conclusions were that the FEM is better suited for this kind of application and that the 3-point method was the fastest method and was easier to use than the 4-pole method.

Keywords: Finite element method; Boundary element method; Numerical methods; Transmission loss; 3-Point method; 4-Pole method; Acoustic silencers

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* Corresponding author. Tel.: + 1-780-492-7031; fax: + 1-780-492-2200.
E-mail address: ken.fyfe@ualberta.ca (K.R. Fyfe).

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1. Introduction

With the ever-increasing computational speed and storage capacity of computers, the use of the finite element method (FEM) and the boundary element method (BEM) in design is growing rapidly. One area that lends itself very well to these methods is the design of silencer systems for noise control. There is much work that has been done for smaller systems such as those used in automobiles and small engines, however, the design of much larger systems (such as the parallel baffle type used for gas turbines and other large industrial machines) is still largely guesswork and empirical extensions of previous results. Due to the large size, difficulties in testing and high costs of these silencer systems, the ability to accurately predict the performance before construction and commissioning would be very beneficial.

To properly predict the performance of a silencer system, many factors need to be involved in the calculation. Geometrical concerns, absorptive material characteristics, flow effects (turbulence), break out noise, self-generated noise, and source impedance all need to be included in the design calculations of insertion loss (*IL*). The main focus of this paper is not to examine each of these areas, but rather to establish the metrics to be used to compare each of these design parameters and to discuss the concerns arising from implementation. Note also, that the methods are applicable only to transmission loss (*TL*) calculations as they are in-duct in nature. *IL* values are preferred for most applications as a final design criteria, however, the *TL* values are still very useful when comparing the performance of one silencer geometry to the next.

Three methods used for calculating *TL* values using both the FEM and BEM are the traditional laboratory method (hereafter referred to as *traditional*), the 4-pole transfer matrix and the 3-point methods. Note that the standards are intended for measuring IL and not TL, but the methods are similar. Each of these methods give similar results for *TL*, however, there are differences in terms of computational time, ease of use, and specific applications. The *traditional* method involves two complete in-duct calculations with and without the silencer installed. The difference between the two resulting frequency response curves is the *TL*. Historically, the 4-pole method has been the preferred computational method and examples of its ability to agree with theoretical and experimental *TL* values are widely documented [1–5]. More recently, however, the 3-point method has been given attention [2,6]. It is easily derived from the fundamental wave equation, and is somewhat less difficult to implement and modify, than the 4-pole method.

As stated before, all three methods will give the *TL* values, however, unlike the *traditional* and 3-point methods, the 4-pole method also gives the pressure and particle velocity values at the inlet and outlet (known as the 4-pole parameters) of the silencer section being evaluated [2]. When designing silencer systems in multiple stages, the 4-pole parameters are necessary for continuity between the sections. However, if the system is being evaluated as one single entity, or if the desire is to evaluate one section by making a change and re-evaluating that one section, then the 4-pole parameters are not required.

The main advantage of the 3-point method is that only one computational run is required while the *traditional*[1] and 4-pole methods require two runs with different geometry and different imposed boundary conditions respectively. Thus, the 3-point method reduces the computational time greatly. It must be noted that there is an improved formulation for the 4-pole method that is faster than the original, while still maintaining the 4-pole parameters [2]. Due to the programming options in SYSNOISE (the numerical program used in this study), however, it was not possible to evaluate the improved 4-pole method and compare it to the other methods.

SYSNOISE is an FEM/BEM based computational acoustics program that allows users to input a geometry, impose boundary conditions, select environmental parameters, and solve the system of resulting equations in one, two or three dimensions [7]. Once the system has been solved, a host of post-processing options are available to determine the various performance characteristics. Using command line code, it was possible to perform the calculations for all three methods, utilizing both FEM and BEM, and in both two and three dimensions.

2. Theory

It is very important to note that the method derivations, and their use with the FEM and BEM are based on plane wave propagation sound sources (i.e. the entire face of the inlet section moving in unison) and an anechoic termination. Situations other than these will need to be addressed on a specific basis.

2.1. Traditional method

The definition of transmission loss is the ratio of the incident sound power to the transmitted sound power. As long as the inlet and outlet regions of the silencer are of the same cross section, and the properties of the fluid (density, temperature) do not change, then the TL can be expressed as:

$$TL = 20\log_{10}\left|\frac{P_i}{P_{\mathrm{ref}}}\quad\frac{P_{\mathrm{ref}}}{P_t}\right| = 20\log_{10}\left|\frac{p_i}{p_t}\right| \tag{1}$$

where: P_i = rms pressure of the incident wave without silencer in place; P_t = rms pressure of the transmitted wave with silencer in place; P_{ref} = reference rms pressure (2×10^{-5} Pa).

This can be simplified to the following equation:

$$TL = SPL_i - SPL_t \tag{2}$$

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[1] Given the limited conditions outlined in the paper and the simple geometries used, the incident sound pressure (SPL_i) for the traditional method can be calculated simply by hand. However, there are instances where this calculation could become difficult, such as varying geometry (tapered section), inclusion of flow, and complex temperature gradients. The method presented in the paper is a general case that can be adapted to include many more complicated scenarios.

where it is understood that SPL_i is obtained without the silencer in place, and SPL_t is obtained with the silencer in place, on the exhaust side of the silencer. This is opposed to the method used to obtain the noise reduction (NR) where both the upstream and downstream values are obtained with the silencer in place. Fig. 1 illustrates the two geometries used to calculate the SPL_i and SPL_t. The SPL_i is calculated with the straight pipe (no expansion chamber) and the SPL_t is calculated with the expansion chamber (no straight pipe). For both geometries, the inlet and outlet sections have the characteristic impedance ($z=\rho c$) boundary condition applied. This models a completely anechoic source and termination. For physical testing, the source and termination ends of the tube are stuffed with absorptive material to minimize reflections and, as close as possible, to mimic an anechoic impedance. Also, for both geometries, the inlet section is given a unit velocity amplitude to model a sound source. All other surfaces are modeled as "acoustically hard" by default.

The *traditional* method is how most standards call for the TL/IL to be measured. The standards usually require the use of an anechoic or reverberation chamber. Note that this method was used as the verification test method for actual tested silencers in the anechoic chamber.

2.2. 4-Pole transfer matrix method

The development of the 4-pole method is well known and can be found in almost any textbook on acoustics or silencer design [1–3,6,8]. Time will then be spent in explaining the resulting formulas and how to implement them in a FEM/BEM modeling scheme.

Using the 4-pole method, a silencer system is evaluated at the inlet and outlet sections by the sound pressure and normal particle velocity [1–3]. Fig. 2 illustrates the measurement locations for a single expansion chamber silencer. The corresponding equations (in matrix format) are:

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} \tag{3}$$

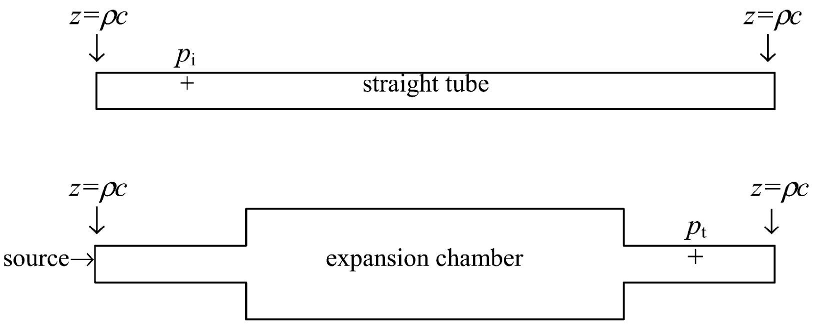

Fig. 1. *Traditional* method measurement points.

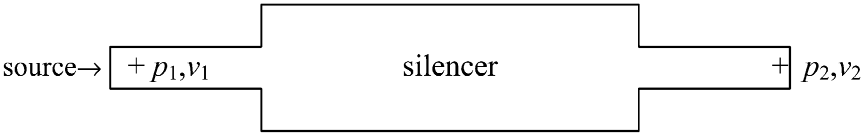

Fig. 2. 4-Pole transfer matrix method measurement points.

where: p_1 = sound pressure at the inlet, p_2 = sound pressure at the exit, v_1 = normal particle velocity at the inlet, v_2 = normal particle velocity at the exit, and

$$A = (p_1/p_2)\big|_{v_2=0,\ v_1=1}, \qquad B = (p_1/v_2)\big|_{p_2=0,\ v_1=1}, \tag{4a, b}$$

$$C = (v_1/p_2)\big|_{v_2=0,\ v_1=1}, \qquad D = (v_1/v_2)\big|_{p_2=0,\ v_1=1}. \tag{4c, d}$$

Finally

$$TL = 20\log_{10}\left(\frac{1}{2}\left|A + \frac{B}{\rho c} + C\rho c + D\right|\right) \tag{5}$$

where, ρc is the characteristic impedance of the acoustic medium.

Once the geometry and environmental conditions (speed of sound and air density) have been set, the next step is to impose the boundary condition of a unit particle velocity at the face of the inlet section ($v_1=1$, $v_2=0$). At this point, the FEM/BEM run can be performed and the values of sound pressure and particle velocity can be determined at the inlet and exit. Parameters A and C can now be calculated using Eqs. (4a) and (4c) and stored for future use.

Next, the second boundary condition of zero pressure at the exit ($v_1=1$, $p_2=0$) can be imposed and the FEM/BEM run is performed again. Once again the pressures and velocities are determined, parameters B and D can be calculated using Eqs. (4b) and (4d). With all four of the parameters known, the TL can be calculated using Eq. (5) at each of the frequencies evaluated.

One important item to note is that when the values of p_1 and p_2 are calculated, they will be in complex format. The numbers MUST remain complex for the duration of the calculations in order to ensure that the phase information is not lost, as would be the case if the complex values were initially converted to absolute values (like when measuring with a real time analyzer).

Another important point to note is that in order to use the 4-pole method, both the pressures and velocities are required. This does not lend itself to easy verification in a laboratory or fully functioning installation. While acoustic pressures are relatively easy to obtain, the velocities are much more difficult to accurately measure. This is in contrast to the *traditional* and 3-point methods where only the pressures are required.

2.3. 3-Point method

Starting from the one-dimensional wave equation, the 3-point method can be derived as [6,8]:

$$p_i = \frac{p_1 - p_2 e^{-ikx_{12}}}{1 - \mathrm{e}^{-i2kx_{12}}} \tag{6}$$

where (referring to Fig. 3): p_i = incoming portion of rms sound pressure wave; p_1 = rms sound pressure at location 1; p_2 = rms sound pressure at location 2; $k = 2\pi f/c$ (wave number); $x_{12} = x_2 - x_1$ (microphone spacing).

Now that the incoming rms pressure values are known, the exiting rms pressure can be obtained and the TL can be calculated simply as follows:

$$TL = 20\log_{10}\left|\frac{p_i}{p_3}\right| \tag{7}$$

where p_3 is the rms sound pressure at point 3. The rms pressure at point 3 can obtained directly since the termination at the exit is given the characteristic impedance ($z = \rho c$). This means that there are no reflected waves and thus the rms pressure at point 3 consists only of the transmitted waves. Note also that the plane wave assumption of the 3-point method limits the upper frequency based on the geometry of the inlet section of the silencer. Further, multi-dimensional versions of this method are being investigated to cover all design cases.

Actual implementation of Eqs. (6) and (7) is quite simple. All that is required is to create the geometry, assign the environmental conditions (speed of sound and air density), impose a unit normal velocity at the inlet section and impose the characteristic impedance ($z = \rho c$) at the exit. The FEM and/or BEM calculations can then be started. In the post-processing stage, the pressures at points 1, 2, and 3 can be calculated and, knowing the distances x_1 and x_2 (or the measurement spacing, x_{12}) and the wave number, k, the TL can be determined.

As noted in the 4-pole method derivation, the calculated pressures will be complex. For the most accurate results, these numbers MUST be kept complex for the duration the calculations. This will ensure that phase information is not lost, as would be the case if the complex values were converted to absolute values.

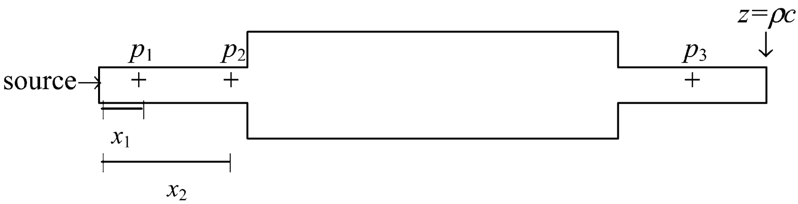

Fig. 3. 3-Pole method measurement points.

2.4. FEM/BEM

SYSNOISE has the ability to perform both FEM and BEM computations. This paper does not contain an in-depth discussion of the intricacies of the FEM and BEM. It is, however, worth spending some time discussing the basic principles for each when applied to solving silencer systems.

The FEM is primarily used when only the interior acoustic field of a geometry is to be computed. Solving purely exterior problems, or coupled interior-exterior problems with the FEM, requires large domains to be modeled with approximate terminating boundary conditions. Alternatively, infinite elements, like the wave envelope method, [9], can be used in these situations. Both strategies require extensive convergence testing to be trusted.

In order to perform the *TL* calculations, the desired region is divided up into a grid of nodes and elements. The fundamental theory behind FEM shows that each element interacts only with the elements directly adjacent to it. With wise node numbering, the result is a banded coefficient matrix for the resulting system of equations, which can be solved faster than a full coefficient matrix. Since the entire acoustic domain is considered for calculation, there also exists the ability to assign different element types, and material properties (such as porosity and density) to different sections of the mesh. This is useful when trying to properly model absorptive materials.

The BEM can be used to compute the interior, exterior, or both fields simultaneously. Unlike the FEM, the BEM only requires the perimeter of the silencer to be divided into nodes and elements and then solved. Also unlike the FEM, each node in the BEM mesh is inter-linked with every other node, which forms a full coefficient matrix. This greatly increases the computational time as the number of nodes increases. It is for this reason that the FEM is preferable for *TL* calculations.

3. Discussion of results

3.1. Theoretical 2D single expansion chamber example

In order to compare the virtues of the *traditional*, the 4-pole and the 3-point methods, with both the FEM and BEM, a single 2D expansion chamber silencer was chosen. Note that this is a 2D problem and, as such, the expansion ratio is 5 and not 25 as would be the case if the silencer was a round, 3D unit. This particular system was chosen because the plane wave theoretical values for *TL* are well established and the geometry is relatively simple [10,11]. Fig. 4 shows the geometry of the modeled silencer (length of 1.2 m and an expansion ratio of 5). The theoretical *TL* curve can be calculated by [11]:

$$TL = 10\log_{10}\left[1 + \frac{1}{4}\left(m - \frac{1}{m}\right)^2 \sin^2 kL\right] \tag{8}$$

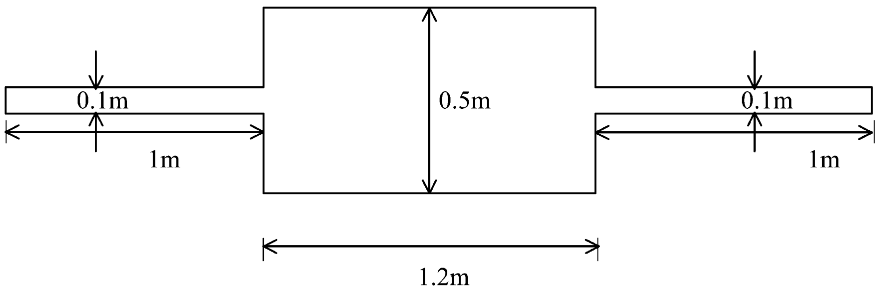

Fig. 4. Single expansion chamber dimensions.

where: m = area ratio of inlet/outlet section to expansion chamber section; k = wave number; L = expansion chamber length.

Fig. 5a shows the FEM mesh created for the geometry as used by SYSNOISE. Fig. 5b is an example of a BEM mesh where each dot represents a node while the lines in between the dots represent elements. The model used for the study was not shown because the large number of elements appear as a single outline when printed on paper, therefore a much simpler model is illustrated. Fig. 6a–c illustrate the results obtained with the *traditional*, 4-pole and 3-point methods respectively for both FEM and BEM. It can be seen that the results for both the *traditional* and 3-point methods overlay each other exactly. The 4-pole FEM results match those of the other methods, and the 4-pole BEM results are also very close to the other BEM results. Note that all three methods follow the theoretical curve up until approximately 340 Hz, which is the point where the theoretical values (based on plane wave propagation) start to lose their validity due to the physical dimensions of the system. It is very important to note that the theoretical values are NOT correct beyond 340 Hz (based on the plane wave propagation assumption) and are shown on the figures beyond 340 Hz to illustrate the point that they do not match with the numerical results, and by how much the differ.

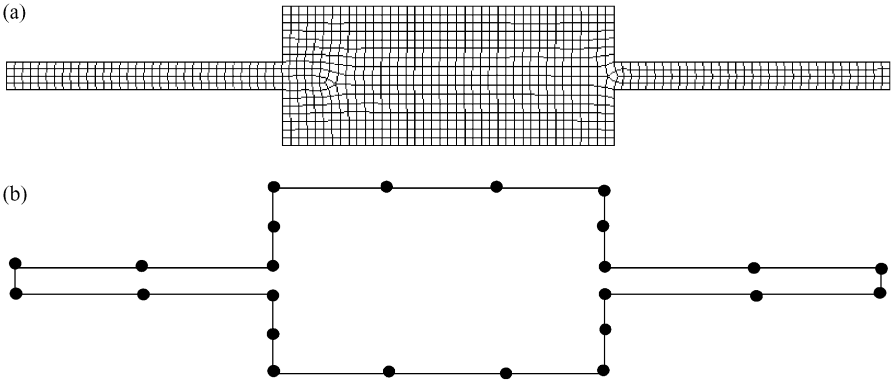

Fig. 5. (a) Finite element mesh (b) example boundary element mesh for single expansion chamber.

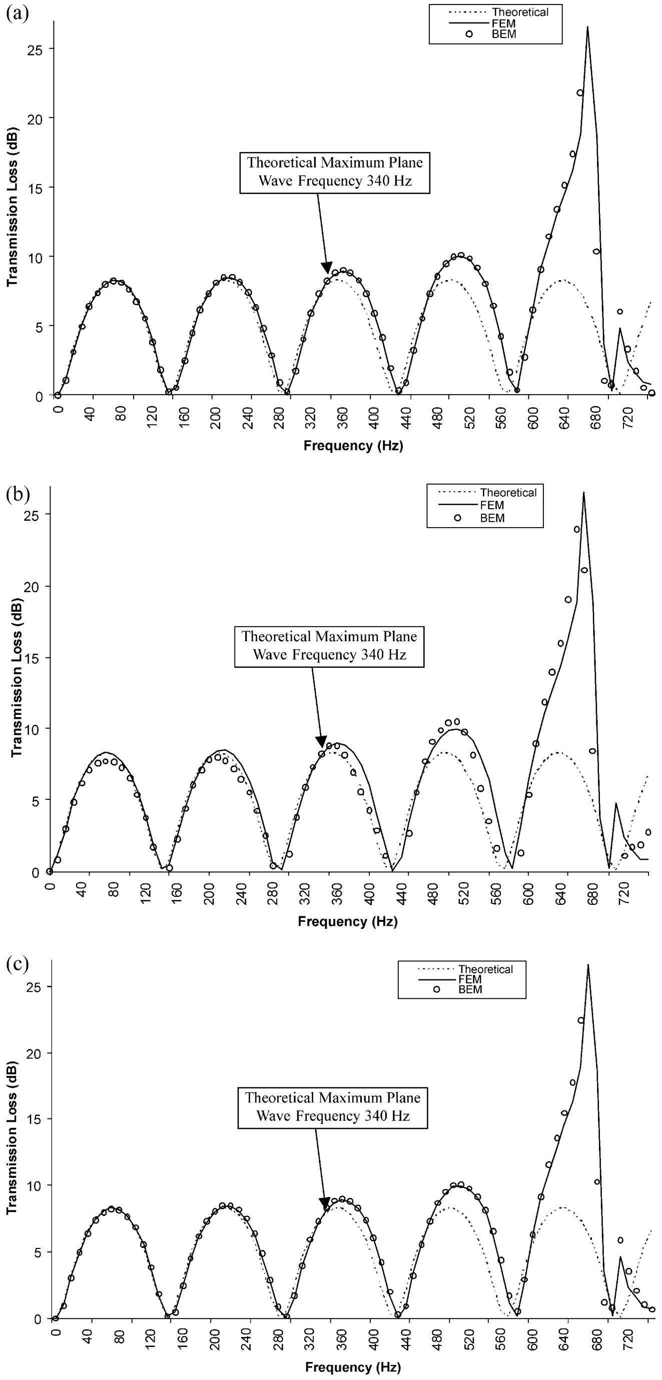

Fig. 6. Transmission loss for single expansion chamber with (a) the *Traditional* method; (b) the 4-pole method; (c) the 3-point method.

In terms of computational time required for solution, the 3-point method was noted to be generally faster than either the *traditional* or 4-pole methods. Again, this was mainly due to the fact that the 3-point method only required one computational "run" while the other two methods required two separate "runs". Also, the FEM was much faster than the BEM due to the fact that the BEM utilizes a full coefficient matrix when solving, while the FEM coefficient matrix is only sparsely populated. It should be noted that a comprehensive study regarding the time parameters was not completed for this paper and, as such, no specific time differences will be reported.

In terms of ease of use and adaptability, the differences between the 3-point method and 4-pole method were minor. The *traditional* method, however, was much more cumbersome to use due to the fact that two separate geometry's were required (one for the straight tube and one for the silenced section) and two separate FEM/BEM runs were required. The 3-point method was a little bit faster to modify and restart than the 4-pole method, however, compared to the overall computational times required, this difference was not an issue. It would be easy to automate the system to make a change, perform the run, record the results, and then start over again.

Overall, the 3-point method was faster and somewhat easier to work with, from a programming standpoint, than the other two methods used. Also, the 3-point method can be implemented in a physical testing scheme much better than the 4-pole method. The particle velocities required for the 4-pole method are difficult to measure, compared to the rms pressures required for the 3-point method.

3.2. Three-dimensional expansion chamber example

To further verify the accuracy and viability of the 3-point method, a comparison was performed with actual measured data from a three-dimensional, round, single expansion chamber silencer with the FEM. The geometry of this silencer is shown in Fig. 7 (note the first part was completed without the stinger) and the results for both measured and calculated values are illustrated in Fig. 8a. It can be seen that the results matched very well (given the small errors possible with the measurement equipment).

In order to decrease the computational time required, the planar-symmetric and axi-symmetric properties of the silencer were exploited. Again, Fig. 8b shows that the full, half, and axi-symmetric models matched exactly with each other. Utilizing

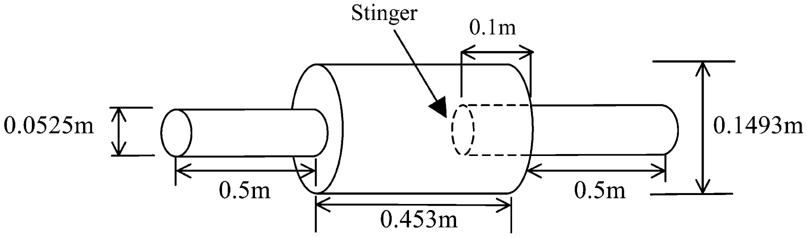

Fig. 7. Full three dimensional expansion chamber dimensions.

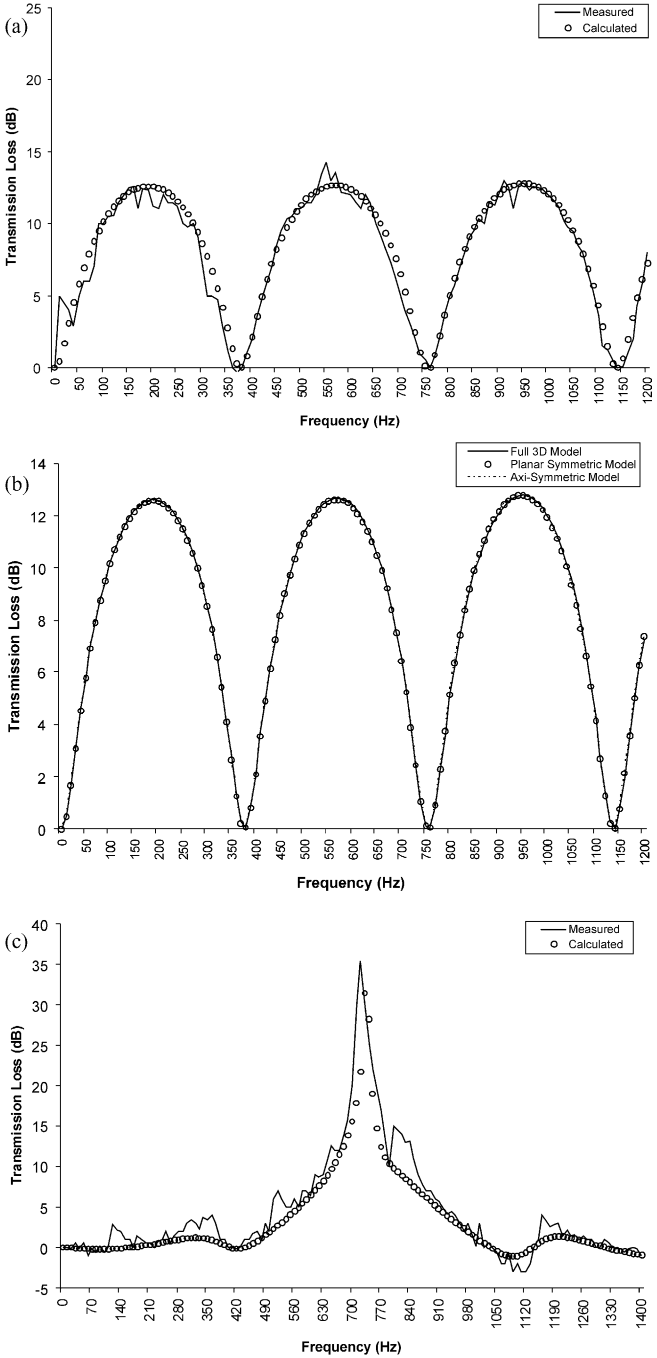

Fig. 8. Transmission loss for three-dimensional single expansion chamber with (a) the 3-point method; (b) the 3-point method in full, half, and axi-symmetric models; (c) the 3-point method and with 10 cm stinger.

lines of symmetry could be applied to more complicated geometry's to greatly reduce the time required and to simplify the model.

Finally, a 10 cm stinger (quarter wave tube resonator) was inserted into the outlet section of the expansion chamber (as shown in Fig. 7) and the modeled and measured values were compared. Once again, Fig. 8c shows that the computed and measured results matched very well. Note that the results shown in Fig. 8c are shown with the *TL* of the expansion chamber subtracted from those of the expansion chamber and stinger so that the end result is the *TL* of the stinger alone.

3.3. Problems/idiosyncrasies encountered

For all three methods there were a few interesting idiosyncrasies associated that are worth mentioning. For example, for each method, varying the length of the inlet and outlet sections of the silencer (varying inlet/outlet lengths from 10% of silencer length to 200% of silencer length) had a very little effect on the calculated *TL*. Only at a few frequencies were any differences realized, and the largest of these differences was never more than 0.5 dB. Higher order modes are likely to be affected at locations very near the junction between the inlet/outlet tubes and the silencing element. Due to the plane wave propagation assumptions made in the method derivations, the measurement points cannot be directly at this junction. In terms of time, the shorter the inlet and outlet sections are, the fewer elements required for the mesh. Therefore, it would be advantageous to make the inlet and outlet sections shorter for no other reason than to save computational effort and time. Through much testing, an inlet/outlet length of 10% of the silencer length was found to be an acceptable minimum length.

The derivations for all three methods evaluated assume plane wave behavior at the inlet and exhaust sections. This is only valid for frequencies up to the point where higher order modes begin to propagate. Beyond this range, the numerical results become increasingly dependent on the choice of measurement location. This was noticed in numerical models as well as in physical system testing. Numerical and physical testing indicated that differences arising from this phenomenon are small (less than 0.5 dB) compared to the TL values obtained. A more detailed analysis of these differences is warranted.

With the *traditional* method, the main concern was reflections from the source side of the geometry. Some of the sound was reflected from the expansion chamber, back to the source, and then reflected from the source back to the expansion chamber. This set up a standing wave that was not realized when the straight tube (with the characteristic impedance at the outlet) was used. When the post silencer measurement, p_t, was taken, this standing wave was present. This resulted in a very large effect on the overall *TL* results (to the point where they were completely different from theoretical). As mentioned before, the way to eliminate this was to simply model the inlet face with an amplitude of vibration and to give it the characteristic impedance so that all of the waves reflected towards it were absorbed and a standing wave was not formed. In actual tests, the way to prevent the formation of standing waves is to install light sound absorbing foam in between the source and the first measurement point (as well as in between the third measurement point and the termination) so that the effects of reflected waves are greatly reduced.

In terms of actual measurements of an in-situ silencer, it is imperative that the values used be complex and that the readings at microphone positions 1 and 2 be simultaneous. Because of this need, it is required that a simultaneous dual channel data acquisition system with FFT calculation capabilities (such as a dual channel FFT analyzer) be used. In doing so, the cross and auto power spectra can be calculated and the pressure value for the incoming wave can be calculated in complex form [8,12,13]. The auto power spectra can then be measured and calculated for the third point and the *TL* can be accurately calculated. It is not the intent of this paper to go into detail regarding the use of the simultaneous dual channel data acquisition system with FFT calculation capabilities and the use of the formulas for obtaining the complex value of the incoming wave. The reader is referred to a paper by Seybert and Ross [13] which discusses the relevant theory in detail.

4. Summary and conclusions

The use of the finite element method and the boundary element method to aid in acoustical engineering design is increasing rapidly. When used in conjunction with the FEM and the BEM, the *traditional*, 4-pole and 3-point methods can be powerful tools for designing acoustical silencer systems. The BEM has been shown to be quite slow when compared to the FEM. It should, therefore, only be used when the modeling demands its flexibility, such as for insertion loss predictions (due to the interior/exterior coupling required). Also, the fundamental differences between the 4-pole method and 3-point method indicate that each one is better suited to certain specific design applications.

The *traditional* method, although as accurate as the other two methods (due to the plane wave assumptions and inlet and outlet boundary conditions), is time consuming and more difficult to implement due to the two separate geometry's required. The 4-pole method is better suited if a cascade of muffler elements is used. The method produces the 4-pole parameters necessary for continuity between adjacent sections. It is slower than the 3-point method and therefore is not recommended for multiple runs or optimization. As mentioned before, there is a modified 4-pole method that has been shown to be just as fast as the 3-point method, but the code required for this method, was not possible to implement with SYSNOISE.

The 3-point method, on the other hand, is just as accurate and easier to use. It is faster than the *traditional* and 4-pole methods and lends itself very well to repeated computational runs for optimization. It does not produce the 4-pole parameters and, as such, the section being evaluated cannot be inter-linked with other sections. In order to perform such an evaluation, all of the sections would need to be created as one large section and then meshed and evaluated. The 3-point method, therefore, is a great tool for evaluating the response of modifying individual parameters such as baffle spacing, absorptive material properties, overall silencer length and width, and effects of multiple small chambers.

5. Future work

There is good agreement between the theoretical and limited experimental results with the 3-point computational method. This encourages further work with full three-dimensional models of reactive and parallel baffle silencer systems to better compare the results.

Also in the future, additional parameters such as absorption, flow and temperature effects should be added to the model to increase the accuracy. The self-generated noise at the exit and the breakout noise from the sides of the system should also be investigated. Ultimately, the *IL* of the system is the desired value, so that the total acoustical impact on an installation can be assessed.

Acknowledgements

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References

[1] Munjal ML. Acoustics of ducts and mufflers. New York: Wiley-Interscience; 1987.

[2] Wu TW, Zhang P, Cheng CYR. Boundary element analysis of mufflers with an improved method for deriving the four-pole parameters. J Sound Vib 1998;217(4):767–79.

[3] Seybert AF, Cheng CYR. Application of the boundary element method to acoustic cavity response and muffler analysis. J Vib Acoust Stress Reliab 1987;109:15–21.

[4] Ji ZL, Selamet A. Boundary element analysis of three-pass perforated duct mufflers. Noise Control Engineering Journal 2000;48(5):151–6.

[5] Selamet A, Ji ZL. Case study: acoustic attenuation performance of circular expansion chambers with extended end-inlet and side-outlet. Noise Control Engineering Journal 2000;48(2):60–6.

[6] Wu TW, Wan GC. Muffler performance studies using direct mixed-body boundary element method and a three-point method for evaluating transmission loss. J Vib Acoust 1996;118:479–84.

[7] SYSNOISE Rev 5.4, LMS International, Interleuvenlann 68, 3001 Leuven, Belgium.

[8] Prasad, M.G. Acoustical modeling of automotive exhaust systems. PHd thesis, Purdue University, 1980.

[9] Cremers L, Fyfe KR. On the use of variable order infinite acoustic wave envelope elements for acoustic radiation and scattering. J Acoust Soc Am 1995;97(4):2028–40.

[10] Beranek LL. Noise and vibration control. New York: McGraw-Hill; 1971.

[11] Bell LH. Industrial noise control. New York: Marcel Dekker; 1994.

[12] Chung JY, Blaser DA. Transfer function method of measuring in-duct acoustic properties. J Acoust Soc Am 1980;68(3):907–21.

[13] Seybert AF, Ross DF. Experimental determination of acoustic properties using a two-microphone random-excitation technique. J Acoust Soc Am 1977;61(5):1362–70.